Solve for x without using division

① $\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 1856$ $\quad x = 928$

② $\frac{1}{4} 4x = \frac{1}{4} 12.8$ $\quad x = 153.2$

③ $\frac{1}{7} \cdot 7x = \frac{1}{7} 4566.8$ $\quad x = 652.4$

$\frac{1}{7} \cdot \frac{7}{1} = 1$

$\frac{1}{13} \cdot \frac{13}{1} = 1$

$\frac{1}{x} \cdot \frac{x}{1} = 1$

$7 \cdot 1 = 7$

$13 \cdot 1 = 13$

$x \cdot 1 = x$

$A \cdot A^{-1} = I$

$A \cdot I = A$

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$A \qquad I \qquad = \qquad A$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 0 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 0 & 7 \end{bmatrix}$$

$2 + 3 \cdot 3 \times 3$

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A \cdot A^{-1}$

$(-3)\ 2a + c = 1$
$\quad\ 4a + 3c = 0$

$-6a - 3c = -3$
$4a + 3c = 0$
$-2a = -3$

$a = \frac{3}{2}$

$2(\frac{3}{2}) + c = 1$

$3 + c = 1$

$c = -2$

$2b + 1d = 0\ (-3)$
$4b + 3d = 1$

$-6b - 3d = 0$
$4b + 3d = 1$
$-2b = 1$

$b = -\frac{1}{2}$

$2(-\frac{1}{2}) + 1d = 0$

$d = 1$

$$\begin{bmatrix} 3/2 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix}$$

determinant → $ad - bc$

If $\det A = 0$, no inverse

$$\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

A

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

A^{-1}

$$\frac{1}{6-4}\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 \\ -2 & 1 \end{bmatrix}$$

Find the inverse

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$A = \frac{1}{6-5}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\frac{1}{1.7 - -2 \cdot -3}\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

$$2x + y = 5$$
$$4x + 3y = 13$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

Coefficient matrix — variable matrix — Answer matrix

$$A \cdot X = B$$

$$X = \begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

2×2 $\quad$ 2×1

$$X = A^{-1} \cdot B$$

$$X = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{aligned} 2x + y - 4z &= 1 \\ 4x - 2y - 2z &= 16 \\ 5x + 3y - 7z &= 12 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & -4 \\ 4 & -2 & -2 \\ 5 & 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 16 \\ 12 \end{bmatrix}$$

$$A \cdot X = B$$

$$X = A^{-1} \cdot B$$

$$X = \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix} = \boxed{\begin{array}{l} x = 6 \\ y = 1 \\ z = 3 \end{array}}$$

Sect. 7.6

#1, 13-17(odd), 26, 41, 42, 44

Sect. 7.7

#1-4, 51

$\det A = |A|$